BIRKHOFF ON DYNAMICAL SYSTEMS

Dynamical Systems. By G. D. Birkhoff. (American Mathematical Society Colloquium Publications, vol. IX.) 295 pp.

In these colloquium lectures the author presents in one volume the ideas and results which he has derived in the last score of years of his work on dynamical theory. It is *modern dynamics* in a sense to which the world has grown accustomed since the thought of Poincaré has become its intellectual heritage. The scope of the work is realized when it is considered in detail.

In Chapter I, the attempt is made to discover what physical meaning there may be in the assumption that the equations of dynamics are Lagrangian. The classical existence and uniqueness theorems for ordinary differential equations are first established. Next, starting with a general system of equations, the author makes physical assumptions of a general nature, with the object of discovering their analytical counterpart. Thus, a conservative system is defined (as in thermodynamics) as one in which the external forces accomplish zero work when the system describes a closed cycle, and from this property a restricted form is obtained from the equations of motion, which includes the Lagrangian as a special case.

Chapter II deals with the variational principles of dynamics. These are derived as usual for the classical systems, as well as for the somewhat more general ones considered in the first chapter. This is also done for Pfaffian systems, a sort of generalization of Hamiltonian systems which the author obtains by replacing in Hamilton's principle

$$\delta\int_{t_0}^{t_1}\left[\sum_{j=1}^{n} p_j q_j' - H\right]dt=0,$$

by

$$\delta\int_{t_0}^{t_1}\left[\sum_{j=1}^{n} P_j p_j' + Q\right]dt=0,$$

(P_j, Q being arbitrary functions of $p_1, \cdots, p_n$; n, even). The transformation theory, and such methods as the ignoration of coordinates, reduction by special integrals, etc., are developed for these systems. The Pfaffian systems have the advantage that they maintain their form under the general point transformation of the coordinates and momenta.

In the neighborhood of a regular point, all differential systems are equivalent under the group of analytic point transformations; they are devoid of invariantive characteristics. The simplest cases in which such characteristics can arise are for the neighborhood of a point of equilibrium, and for the neighborhood of a periodic solution, which can be reduced to a generalized point of equilibrium. Our attention is thus naturally directed to such motions, which are, moreover, of considerable physical importance. In Chapter III, these motions are considered, and a complete treatment of those of their invariantive properties which are of a purely formal nature is given in the general case (where there are no linear relations with integral coefficients between the