# ON A DECOMPOSITION OF A QUATERNARY QUADRATIC FORM*

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For a binary quadratic form it is very easy to prove that it can be decomposed in a unique way into a sum of a form of the type $e(x^2+y^2)$ and another which by an orthogonal transformation can be reduced to $q(x^2-y^2)$.† It is the purpose of this note to prove an analogous theorem for quadratic quaternary forms, namely, that *such a form can be presented as the sum of three forms:*

(a) *a form of the type* $e(x^2+y^2+z^2+t^2)$,

(b) *a form which by an orthogonal transformation can be reduced to* $q(x^2+y^2-z^2-t^2)$, *and*

(c) *a square of a linear form* $(ax+by+cz+dt)^2$.

*The decomposition in general is unique; an exceptional case is indicated below.*

It seems that the decomposition is interesting in itself but it gains in interest because of possible application in physics. Translated into tensor language the theorem may be stated as follows: a symmetric tensor of rank two in four-dimensional space may be presented, in general in a unique way, as the sum of a tensor of the type of a hydrodynamical tensor $\delta g_{ij}+\rho u_i u_j$, where $\delta$ corresponds to pressure, $\rho$ to density and $u_i$ is a unit vector, the four-dimensional velocity; and an electromagnetic stress-energy tensor.‡ If the theorem were true in pseudo-euclidean space, it would mean that the contracted Riemann tensor, set equal to the sum of the two above tensors, determines (in general, uniquely) the field of matter and the electromagnetic field. However, the situation in pseudo-euclidean space is more complicated and the theorem there is not true without modifica-

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† These two types correspond, if the forms are interpreted as second differential forms of surfaces, to a sphere and to a minimal surface respectively.

‡ For a proof that this tensor is of the type (b), see Proceedings of the National Academy of Science, vol. 10 (1924), p. 126; or Transactions of this Society, vol. 27 (1925), p. 117.