

tion of this lemma to the case where the coefficients of the system (5) and the initial data (6) depend on a parameter. The interval  $(0, 1)$  of course can be replaced by an arbitrary interval  $(a, b)$ , the initial values  $\psi'(0)$ ,  $\psi''(0)$  may be made distinct and the whole theory can be extended to systems of infinitely many equations, under suitable restrictions upon the matrices and vectors involved.

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## ON THE NUMBER OF APPARENT MULTIPLE POINTS OF VARIETIES IN HYPERSPACE

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By an apparent point of multiplicity  $s$  on a variety in  $r$ -space we mean a line which passes through a given point in  $r$ -space and meets the variety in  $s$  distinct points. In order that the number of such apparent  $s$ -fold points on a variety  $V_x^m$  of order  $m$  and of  $x$  dimensions be finite, we must have  $r = st + 1$ ,  $x = (s - 1)t$ , where  $t$  is the number less one of the hypersurfaces intersecting in the variety. In other words, the number of apparent  $s$ -fold points on a  $V_{(s-1)t}^m$  which is the intersection of  $t + 1$  hypersurfaces in  $S_{st+1}$  is finite. It is the purpose of this paper to determine this number and also to determine its upper and lower limits.

We shall use the symbols  $H_s^{(r)}$ ,  $\bar{h}_s^{(r)}$  to denote respectively the maximum and minimum number of apparent  $s$ -fold points that a  $V_{(s-1)t}^m$  of any order  $m$  in  $S_r[r = st + 1]$  can have, and the symbol  $h_s^{(r)}$  to denote the number of those that a  $V_{(s-1)t}^m$  of order  $m = n_1 n_2 \cdots n_{t+1}$ , which is the complete intersection of  $t + 1$  hypersurfaces of orders  $n_1, n_2, \cdots, n_{t+1}$  respectively, ordinarily has. Thus if  $s = 2$ ,  $t = 1$  and therefore  $r = 3$ , a curve  $C^m$  in  $S_3$  can have at most

$$(1) \quad H_2^{(3)} = (m - 1)(m - 2)/2,$$

and at least

$$(2) \quad \bar{h}_2^{(3)} = m(m - 2)/4, \quad \text{for } m \text{ even,}$$

and

$$(2') \quad \bar{h}_2^{(3)} = (m - 1)^2/4, \quad \text{for } m \text{ odd,}$$