THE SIMPLEST INVOLUTORIAL TRANSFORMATION CONTAINED MULTIPLY IN A LINE COMPLEX*

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Involutorial birational transformations contained multiply in a linear line complex are interesting, as they furnish the simplest examples of involutions that are probably irrational. The following example is the simplest possible case of such a transformation; it possesses features that are characteristic, and may itself be irrational.†

Let $(0, 0, z_3, z_4)$ be a variable point on the line $x_1=0$, $x_2=0$, and $\lambda_1 x_3^2+\lambda_2 x_4^2=0$ an involution I_2 of pairs of planes of a pencil, the axis being skew to the first line. Let us suppose that the point (z) and the planes (λ) are connected by the relation $z_3\lambda_1^2+z_4\lambda_2^2=0$. A point (y) of space determines the pair of planes $x_3^2y_4^2-x_4^2y_3^2=0$ of the pencil; hence $z_3=y_3^4$, $z_4=y_4^4$. A point (x) on the line joining (y) to (z) has coordinates of the form $\rho x_1=\sigma y_1$, $\rho x_2=\sigma y_2$, $\rho x_3=\sigma y_3+\tau z_3$, $\rho x_4=\sigma y_4+\tau z_4$. This line meets the plane conjugate to that determined by (y) in (y'), corresponding to $\sigma=y_3^3+y_4^3$, $\tau=-2$.

The points (y), (y') are therefore associated in an involutorial birational transformation, the equations of which have the form

$$(I_4)\qquad \begin{cases} \rho x_1' = (x_3^3+x_4^3)x_1, \\ \rho x_2' = (x_3^3+x_4^3)x_2, \\ \rho x_3' = (x_4^3-x_3^3)x_3, \\ \rho x_4' = (x_3^3-x_4^3)x_4. \end{cases}$$

The transformation I_4 is contained doubly in the special linear line complex, the axis of which is $x_1=0$, $x_2=0$ in the sense that each line of the complex contains two pairs of conjugate points in I_4. Every plane through the axis is transformed into

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† Another involution of order 2 that is probably irrational is described in the Giornale di Matematiche, vol. 61 (1923), pp. 125–128.