NON-EUCLIDEAN GEOMETRY, A RETROSPECT*

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1. *Introduction.* Non-euclidean geometry had its origin in the unwearied attempts of mathematicians during 2000 years to free the *Elements* from its one notorious blemish, namely, the postulate or axiom relating to parallel lines.

Wallis (1693), Saccheri (1733), Lambert (1786), Legendre (1794), Schweikert (1807), Wachter (1817), and Taurinus (1826) are noteworthy forerunners, but the first systematic and rigorous development of the subject to be published was a series of five papers entitled *On the principles of geometry*, which appeared during 1829–30 in the Kasan Messenger. Their author was an unknown Russian mathematician, Lobachevsky. The present year 1929 may be regarded as rounding out the first century of this new science. What has it accomplished in this time? I propose to answer this question, paying attention not so much to concrete results obtained, as to the basal ideas which have made its phenomenal progress possible.

The early history of our subject is too well known to require more than a few words. We must mention, however, that Gauss was already in full possession of Lobachevsky's results, although he permitted nothing to reach the ears of the public. The Hungarian Bolyai had also broken through the barriers of euclidean geometry independently. His treatise, through no fault of his own, did not appear till 1832.

The method used by these geometers was the synthetic method of Euclid, with a slight mixture of trigonometry, analytic geometry and the calculus. It was adequate to establish the existence of a non-euclidean geometry, but new methods and ideas were necessary for further progress. These came with a bound.

Riemann's Habilitationsschrift *Ueber die Hypothesen welche der Geometrie zu Grunde liegen*, read before the Philosophical Faculty of the University of Goettingen in 1854, but first

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