SHORTER NOTICES

Einleitung in die Mengenlehre. By A. Fraenkel. 3d ed. Berlin, Springer, 1928. 14+424 pp.

Mengenlehre. By E. Kamke. (Sammlung Göschen, No. 999.) Berlin and Leipzig, de Gruyter, 1928. 159 pp.

Books on the theory of aggregates all seem to present much the same approach to the subject. They begin with Cantor's definition of an aggregate and then build up a theory of cardinal and ordinal numbers. It seems to the reviewer that one of the main features of the aggregate theory is its extremely careful avoidance of any reasoning which is not clear cut and rigorous. For a student entering for the first time on rigorous thinking, the aggregate theory seems almost uselessly careful. But should a student have his first dip into rigor in this field? As a matter of fact most students get an introduction to rigor in a course in real variable. Then why, in a course on aggregate theory; and why, in the texts on the subject, is it not possible to put down a system of axioms which is considered sufficient for the purpose in hand and to proceed by indisputable steps to the erection of the structure of the theory? The two books about to be reviewed do not do this. The first, by Fraenkel, discusses the axioms very carefully, but not until the very end of the book! The second even proves the well ordering theorem without any mention of Zermelo's Axiom.

The treatise by Fraenkel on the theory of aggregates is now one of the finest. The book has been made much clearer by the definite statements of theorems in italics, which was not done in the previous editions.

This book is divided into five chapters: I. Foundations and Cardinal Numbers, II. Operations with Cardinal Numbers, III. Order Types and Ordinal Numbers, IV. Attacks on the Foundations and their Consequences, V. The Axiomatic Structure of the Theory of Aggregates and the Axiomatic Method.

The first three chapters cover the usual ground quite clearly except that what is assumed is sometimes a bit in doubt because no axioms are presented. The book contains problems which make it very suitable as a text for graduate students.

The fourth chapter has been enlarged to twice its former size and presents a most interesting discussion of Brouwer's new work and of Russell's work.

The fifth chapter discusses a system of axioms and Hilbert's theory of logic and metamathematics.

Not as much attention is paid in the book to the application of the theory to function theory or point sets as might be in a text of its size; it is a text on the theoretical side of the subject only.

In matters of printing and arrangement both author and publisher are to be complimented.

Kamke's little book is a very convenient summary of the subject of abstract aggregate theory. Its four chapters are concerned with, first, the generalities of the subject; second, cardinal numbers; third, ordered sets and ordered types; fourth, well ordered sets and their ordinal numbers.