

they do not distinguish. The first of these expressions is most naturally interpreted as a proposition, p , which asserts, " p is false"; so that we have a proposition directly about itself, in connection with which a genuine vicious circle arises. But the authors do not consider this case; instead, they take the most natural interpretation of the second of the foregoing expressions, according to which it is to be rendered: "There is one and only one proposition which I am asserting, and it is false." Clearly, this is often true; but it cannot be true if I assert it, nor can it be false; and yet a vicious circle does not arise, since all that follows is that I never do assert it. Nevertheless, the situation here in question is just as objectionable as the occurrence of a vicious circle, since it leads to the conclusion that a logically possible state of affairs is in fact impossible.

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NÖRLUND ON FINITE DIFFERENCES

Leçons sur les Équations Linéaires aux Différences Finies. By N. E. Nörlund. Paris, Gauthier-Villars, 1929. vi+153 pp.

The theory of finite difference equations arose in investigations by Lagrange and Laplace; but it is only recently that the properties of the solutions of these equations have been developed with any considerable detail. The modern researches on this subject have been inaugurated by Poincaré and Pincherle. One owes to Poincaré a remarkable theorem on the manner in which the solutions of a linear homogeneous equation in finite differences behave for very large values of the variable. This theorem has been the point of departure of several investigations. In his preface the author says: "In recent years the theory of finite difference equations has been developed by a large number of authors, among whom may be mentioned G. D. Birkhoff, H. Galbrun, E. Hilb, E. Bortolotti, O. Perron, R. D. Carmichael, J. Horn, K. P. Williams, A. Guldberg and G. Wallenberg. [To this list, of course, should be added the name of Nörlund himself.] The subject is too vast for it to be possible to give here an exposition of all the results obtained. The aim of this book is to put in evidence the essential properties of the solutions of linear homogeneous equations, on the one hand by aid of their development in factorial series, on the other hand by aid of certain methods of successive approximations due to G. D. Birkhoff and R. D. Carmichael." Chapters I-IV are devoted to a single linear equation, different hypotheses relative to the coefficients being made in the different chapters. Chapters V-VI treat similar problems for a system of linear equations.

The first chapter is devoted to general properties of linear equations, such as the existence theorems which are readily proved, adjoint equations, equations with second member, and the reduction of the order of an equation by means of known solutions. A large part of this chapter is elementary and is devoted to the preliminaries of the general theory. There is, however, an existence theorem of considerable importance, based on hypotheses of a broad general character. In the fifth chapter one finds a treatment of precisely similar ques-