CLASSES OF DIOPHANTINE EQUATIONS WHOSE POSITIVE INTEGRAL SOLUTIONS ARE BOUNDED*

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1. *Introduction.* If upper bounds have been found for the positive integral solutions of a diophantine equation, the problem of obtaining all such solutions is reduced to making a finite number of trials. It may therefore be of interest to note certain cases where upper bounds are given by simple algegraic processes. Hereafter the term *solution* will always mean a solution in positive integers.

Our starting point is the observation that if $P(t)$ is a polynomial in t, then all positive values of x that satisfy the inequality $P(1/x) \geqq 0$ are bounded if (and only if) the term of lowest degree in $P(t)$ has a negative coefficient.

2. *A Type whose Solutions are always Bounded.* Every algebraic diophantine equation in n variables $x_1, x_2, \cdots, x_n$ can be thrown into a form where the right side is zero and the left side is a polynomial in the reciprocals of the x's. When this has been done, the first type here to be considered is the following:

$$F\left(\frac{1}{x_1}, \frac{1}{x_2}, \cdots, \frac{1}{x_n}\right) - k = 0, \tag{1}$$

where F is a polynomial all of whose coefficients are positive, while k is a positive constant, and $F(0, 0, \cdots, 0) = 0$.

The positive integral solutions of every equation of type (1) *are bounded.*

To prove this statement, and to show how to obtain bounds for the solutions, let us first consider a solution such that $x_1 \leqq x_2 \leqq \cdots \leqq x_n$. We shall then have

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