CONTINUOUS CURVES IN WHICH EVERY ARC MAY BE EXTENDED*

BY W. L. AYRES†

1. *Introduction.* An arc α belonging to a point set K is said to be a *maximal arc* of K if there is no arc which belongs to K and contains the arc α as a proper subset. S. Mazurkiewicz‡ proved in 1921 that every arc of a bounded acyclic continuous curve§ M is a subset of a maximal arc of M. However there are many types of continuous curves in which every arc of a continuous curve is a subset of a maximal arc and which are not acyclic; and in a recent paper|| we have given a necessary and sufficient condition in order that a continuous curve have this property. In this note we shall treat the opposite problem of finding under what conditions no arc is a subset of a maximal arc. We shall say that an arc α of a point set K is *extendible* in K if there exists an arc β of K of which α is a proper subset. If X and Y are the end points of α and Y is an interior point of β, then we say that α is *extendible in K in the direction XY*. And if there exists an arc β of K containing α and such that both end points of α are interior points of β, then we say α is *extendible in K in both directions*. In this note we shall give two necessary and suffi-

---

* Presented to the Society, December 27, 1928.

† National Research Fellow in Mathematics.

‡ *Un théorème sur les lignes de Jordan*, Fundamenta Mathematicae, vol. 2 (1921), pp. 119–130. See Lemma 13, p. 129.

§ A *continuous curve* is a closed connected, and connected im kleinen point set. A continuous curve is *acyclic* if it contains no simple closed curve. It must be noticed that under these definitions a single point is a continuous curve and is acyclic. However, to avoid trivial cases, we will assume that the continuous curve M, which is mentioned repeatedly, contains more than one point.

|| W. L. Ayres, *Conditions under which every arc of a continuous curve is a subset of a maximal arc of the curve*, Mathematische Annalen, vol. 101 (1929), pp. 194–209.