the same manner, we obtain as the image of V_k^μ a $V_k^{\mu^2}$ which is of the same nature as that obtained by means of the r^2-ic transformation as the image of an S_{r-1}.

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ON SOME FUNCTIONS CONNECTED WITH $\phi(n)$

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Let $\phi(n)$ denote, as usual, the number of numbers not greater than and prime to n. Let $N(x)$ be the number of distinct numbers less than x, which can be the ϕ function of some number; and let $R(n)$ be the number of solutions of the equation

$$n = \phi(x),$$

n being given. The object of this note is to prove some results concerning the magnitude of $N(n)$ and to apply them to prove that

$$\overline{\lim_{n=\infty}} R(n) = \infty .$$

Since there is no reference to such results in Dickson's *History of the Theory of Numbers*, I believe that the last result in particular is new.

THEOREM I. *We have*

$$N(n) > \frac{a \cdot n}{\log n},$$

where a is a constant.

PROOF. For each prime p, $\phi(p) = p - 1$; hence, if we denote by $\pi(n)$ the number of primes not exceeding n, then

$$N(n) \geqq \pi(n).$$

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* I take this opportunity to express my deep gratitude to K. Ananda Rao for his invaluable guidance and encouragement.

This paper was read before the conference of the Indian Mathematical Society held in December 1928.