A CERTAIN BIRATIONAL TRANSFORMATION OF ORDER r^2 BETWEEN TWO r-SPACES IN AN $(r+1)$-SPACE

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In this paper we present a birational transformation of order r^2 between two r-spaces S_r and S_r' in an $(r+1)$-space S_{r+1}. If $r=2$, we have the well known quartic birational transformation between two planes of an S_3 obtained by intersecting with the two planes the bisecants of a twisted cubic curve in S_3. A point of one plane and a point of the other are said to be corresponding points if they lie on the same bisecant of the cubic curve. It is our object to generalize this construction to hyperspace.

For this purpose we let two r-ic hypersurfaces (necessarily ruled) intersect in a composite manifold composed of an M_{r-1}^n of order $n=r(r+1)/2$ and an $M_{r-1}^{n'}$ of order $n'=r(r-1)/2$. The former component manifold, M_{r-1}^n, will have just one apparent r-fold point* if it meets the latter, M_{r-1}, in an $(r-2)$-dimensional variety of order $r(r^2-1)/3$. This implies that M_{r-1}^n and $M_{r-1}^{n'}$ are such that a general S_3 meets them in two curves C^n and $C^{n'}$, respectively, having $r(r^2-1)/3$ points in common and that C^n has $r(r^2-1)\cdot(3r-2)/24$ and $C^{n'}$ has $r(r-1)(r-2)(3r-5)/24$ apparent double points. It also implies that a general S_4 meets them in two surfaces having respectively $r(r+1)(r-1)^2(r-2)^2/48$ and $r(r-1)(r-2)^2(r-3)^2/48$ apparent triple points. The number of apparent $(k-1)$-fold points on their intersections with an S_k can also be found.†

As we are going to make use of M_{r-1}^n‡ to obtain an r^2-ic

* By an apparent r-fold point of a V_{r-1} in S_{r-1} we mean a line passing through a general point of S_{r+1} and meeting V_{r-1} r times.

† See B. C. Wong, *On the number of apparent triple points of surfaces in space of four dimensions*, this Bulletin, vol. 35, No. 3, pp. 339–343.

‡ There are in S_{r+1} many $(r-1)$-dimensional varieties of order m $[2r-1\leqq m\leqq r(r+1)/2]$ that have just one apparent r-fold point but the M_{r-1}^n here described is the only one that offers the desired transformation.