ON ORDINARY RESTRICTED EXTREMA IN CONNECTION WITH POINT TRANSFORMATIONS

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1. *Restricted Minima in a Plane.* The question which I shall examine concerns the maximum or minimum of $f(x, y)$, when the variables x and y are not independent, but subjected to the condition

$$g(x,y) = 0, \tag{g}$$

so that the point (x, y) must describe the curve represented by the preceding equation.

This is what was called, formerly, a *relative* extremum, but what I proposed to call a *restricted** extremum, because the term "relative" is used with another meaning. Indeed, the extrema which we consider, as every extremum treated by the methods of differential calculus, are *relative*, that is, they are extrema only in comparison with neighboring values of f.

The first part of the question, namely, the investigation of stationary values of f on the curve (g), is quite classic. It is usually solved, with the help of the Lagrange multiplier l, by means of the simultaneous equations

$$p + lp_1 = 0, \qquad q + lq_1 = 0, \tag{1}$$

where p and q are the partial derivatives of f, and p_1 and q_1 are those of g. However, we shall use here the condition with l eliminated, that is,

$$j = pq_1 - qp_1 = 0, \tag{j}$$

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* The author informs the editors that he, together with David Hilbert and others, had previously agreed that a change of nomenclature is desirable, to replace the older term relative extrema. The agreement is to use "*extremum libre ou lié*" in French, and "*freies oder gebundenes Extremum*" in German. The author had used "*bound*" in English, and the editors have suggested the word "*restricted.*" THE EDITORS.