# NOTE ON A CERTAIN TYPE OF PARABOLA*

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In the development of the geometry of the complex domain, there has been a tendency to pass over problems of a relatively elementary character, despite their central position and importance. It is the purpose of this note to call attention to one of these problems, namely that of the classification of complex conics with respect to the group of complex rigid motions. A systematic study of this problem brings to light two *special* types of non-degenerate conics: the non-degenerate central conics which contain just one circular point at infinity, and the non-degenerate parabolas which are tangent to the line at infinity at a circular point. These special conics have already been considered, from a different point of view, in this Bulletin.†

It is not our intention to go into detail, either in connection with the general problem of classification or in consideration of the special conics. We propose merely to discuss certain particularly striking properties of the special parabolas.

It is evident that the complex conic

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0, \tag{1}$$

$A$, $B$, $C$ not all zero, is a special parabola if and only if

$$B^2 - 4AC = 0, \quad A + C = 0, \quad \Delta \neq 0, \tag{2}$$

where $\Delta$ is the discriminant of (1).

Since a special parabola has neither focus nor directrix, the question of its eccentricity cannot be handled directly. We may, however, establish an equation determining the

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† E. V. Huntington and J. K. Whittemore, *Some curious properties of conics touching the line at infinity at one of the circular points*, vol. 8 (1901–02), pp. 122–124.