

SHORTER NOTICES

Applications of the Theory of Elliptic Functions to the Theory of Numbers.

By P. S. Nazimoff. Translated from the Russian by Arnold E. Ross.
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The original of this remarkable book was presented as a magister thesis in Russia and was printed in Russian at Moscow in 1884. Abstracts of certain parts of it were published shortly after its Russian appearance, both in the *Fortschritte* and in a prominent French journal. Neither abstract did the book justice. That this assertion of fact is so can be seen from the following. Had an adequate account of the contents of this book appeared at the time of its publication, much later work in the theory of numbers of representations of integers in special quadratic forms need never have been published. For example, in 1914, Bulyguin, using the methods of the first few chapters of Nazimoff's book, gave a complete solution, by comparatively elementary means, of the determination of the numbers of representations of integers as sums of any even number of squares. This work itself, like the neglected Nazimoff's, seems to have escaped the notice of the professional abstractors. As late as 1919, G. Humbert, in a letter to the present reviewer, inquired what had become of this most remarkable work of Bulyguin; it was already in print. Episodes such as this but emphasize the lamentable ignorance under which most of us labor concerning Russian work in the theory of numbers. Reverting for a moment to the question of representations as a sum of an even number of squares, we may state that many recent and elaborate investigations were out of date before they were printed, on account of the work of the neglected Russian school.

In the present translation only those parts of Nazimoff's book have been presented that contain matter which was original at the time of publication and a great part of which, in the light of the above remarks, is still as fresh as it was when published. The title sufficiently describes the contents of the work. Nazimoff made the first systematic attempt to obtain everything arithmetical that is implicit in the theory of elliptic functions. That such a vast undertaking appears only rough hewn in the final product, is not to the author's discredit. Precisely where his work is least finished is the place where it is most suggestive. The problem of discovering everything that is implied for the theory of numbers by the multiply periodic functions is a well-put and solvable one; this book took the first significant step toward a solution. It is worthy of the attention of all those interested in the algebraic applications of analysis to the theory of numbers—which is quite a different thing from the analytic theory of numbers.

For those interested, the general problem which will be suggested by a reading of pages 138–144 of the present translation, is worthy of the closest attention. A complete solution of the obviously implied problem, extended to functions of any number of variables, contains the solution of any