PICARD ON PARTIAL DIFFERENTIAL EQUATIONS

Leçons sur quelques Types Simples d'Equations aux Dérivées Partielles avec des Applications à la Physique Mathématique. By Émile Picard. Paris, Gauthier-Villars, 1927. i+214 pp.

This book forms the first volume of a new mathematical series, *Cahiers Scientifiques,* edited by G. Julia. From the preface to volume 3 of the series, one learns with regret that Picard has abandoned the project of completing volume 4 of his *Traité d'Analyse,* which was to give the theory of partial differential equations. Instead, the subject matter of this volume will be presented in several volumes of *Cahiers Scientifiques.* The book under review contains lectures given at the Sorbonne in 1907 and repeated, with some additions, in 1925.

The first four lectures deal with the simplest and best known of the equations of parabolic type, namely Fourier's equation

$$\partial u/\partial x = \partial^2 u/\partial y^2.$$

Lecture 1 begins with the existence theorem, shows that non-analytic solutions are possible for analytic initial data, and derives Fredholm's example of a power series whose circle of convergence is a natural boundary. Lecture 2 derives the fundamental solution, shows how this is used to solve the boundary problem $u=f(x)$ for $t=0$, and as an application gives Weierstrass' original proof of his theorem on the approximation to a continuous function by polynomials. Lecture 3 applies the preceding results to various boundary problems in heat conduction, and Lecture 4 gives an account of Lord Kelvin's investigation of the propagation of an electric current along a cable.

Lectures 5 to 11 deal with various aspects of the theory of integral equations. In Lecture 5, a brief account of the Fourier integral is given and applied to an integral equation which was set up in Lecture 3. Lecture 6 deals with the properties of the integrals

$$\int_0^\infty \cos xy\phi(y)dy \text{ and } \int_0^\infty e^{-xy}\phi(y)dy$$

considered as functions of x. In Lecture 7, an integral equation of the first kind is set up for the solution of Dirichlet's problem (in two dimensions) by a potential of a single layer on the boundary curve, and the main results of Fredholm on integral equations of the second kind are enumerated.

In Lecture 8, the singular integral equations of the second kind

$$f(x) + \lambda \int_0^\infty \cos x\xi f(\xi)d\xi = \phi(x)$$

and

$$f(x) + \lambda \int_{-\infty}^\infty e^{-|x-\xi|} f(\xi)d\xi = \phi(x)$$