DE VRIES ON FOUR DIMENSIONS

Die vierte Dimension, eine Einführung in das Vergleichende Studium der Verschiedenen Geometrien. By Dr. H. K. de Vries. Translated into German from the second Dutch edition by Frau Dr. Ruth Struik. Wissenschaft und Hypothese, XXIX, Leipzig and Berlin, Teubner, 1926. ix+167 pp.

Although there is no mention of non-euclidean geometry in the title, nor even on the title-page, nearly half of the book is devoted to it. In fact, we have here two books put together under the title of the first. The books are independent: either could have been put first or published alone. Only in one place in the second (pages 139–141) is there introduced a little of the geometry of four dimensions, and at the very end there is an outline of the rise of this geometry, a little more than two pages, that might have been put at the end of the first part.

The treatment of the book is not strictly mathematical. We have rather a popular exposition, although considerable mathematical detail is suggested in the explanations, giving some idea of the way in which these subjects have been developed as well as the mere results of their development.

A small book, not too condensed, must leave out many things. Thus we find nothing about hyperprisms, hyperpyramids, hypercylinders, and hypercones, and nothing about hypervolume. Some might have chosen these and given only a paragraph to the regular polytopes or hypersolids. Many details of the non-euclidean geometry are presented very briefly. But the choice of material is in general excellent, and the book is well planned to impart some knowledge of these two branches of mathematics.

In both parts of the book much is said about the historical development of the subjects considered, about their usefulness, and about the frame of mind in which the reader should study them, and many interesting remarks are thrown in. Thus of the value of extending the chain of dimensions beyond the third we are told (page 22) that the notion of value is in high degree relative, and what is of inestimable value for one man leaves another entirely cold. But for the mathematician and for everyone who has an interest in scientific thought, it is of great value to know that a four or five or n dimensional geometry is possible, and the mathematician will not be satisfied to know that it is possible, but will want to know more exactly about it, as always when a new field of investigation appears.

At the beginning the relations of the simpler elements, lines, planes, and hyperplanes, their determinations and intersections, parallelism and perpendicularity, and the various kinds of angles formed by them, are gone into very thoroughly and carefully, and this study is the best sort of preparation even for a superficial understanding of the more complicated figures and relations of hyperspace.

The notion of point-value of a space is most helpful, for the dimension-number d is 1 less than w, the number of points necessary to determine a given space, and some formulas are obtained more easily by consider-