SINGULAR MANIFOLDS AMONG THOSE OF AN ANALYTIC FAMILY*

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1. *Exceptional Occurrence of Singular Manifolds.* This note is concerned with the following theorem.

Let R denote a closed region, consisting of an open continuum of the space of the n complex variables $z_1, z_2, \cdots, z_n$, *together with its boundary points. Let the functions* $F_i(z_1, z_2, \cdots, z_n)$, $i=1, 2, \cdots, m$, $m \leqq n$, *be analytic at all points of R. Let* M_k *denote the matrix*

$$(M_k) \qquad \left\| \frac{\partial F_i}{\partial z_j} \right\|, \qquad \begin{array}{l} (i = 1,2, \cdots, k), \\ (j = 1,2, \cdots, n). \end{array}$$

We assume that M_m *is of rank m at some point of R.*

Consider the manifold defined by the equations

$$(A) \quad F_1(z_1,z_2, \cdots, z_n) = c_1, \cdots, F_m(z_1,z_2, \cdots, z_n) = c_m,$$

where $c_1, c_2, \cdots, c_m$ *are complex constants.*

For all but a finite number of values of c_1 *the manifold defined by the first equation (A) contains no points in R at which the rank of* M_1 *is less than* 1.

If $c_1, c_2, \cdots, c_k$ *have been chosen so that the matrix* M_k *is of rank k at every point in R on the manifold defined by the first k equations (A), then for all but a finite number of values of* c_{k+1} *the manifold defined by the first* $k+1$ *equations (A) contains no points in R at which the rank of the matrix* M_{k+1} *is less then* $k+1$.

Thus, if $c_1, c_2, \cdots, c_m$ *are chosen in order, each avoiding a certain finite set of values, the manifold defined by the equations (A) will have no singular points in R.*†

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† As far as I know, a proof of this general theorem has not been published. Birkhoff and I gave it for the case in which the functions F_i are polynomials (Transactions of this Society, vol. 23 (1922), pp. 97–98), and