sets M_1 and M_2 in R, there exists at least one critical point of $f(x_1, \cdots, x_n)$ in R that does not belong to a minimal set of $f(x_1, \cdots, x_n)$.

COROLLARY 2. *If $f(x_1, \cdots, x_n)$ is restricted so that each of its minimal sets contains a single point, Theorem 3 becomes Bieberbach's form of the minimax principle.**

HARVARD UNIVERSITY
and
UNIVERSITY OF CALIFORNIA AT LOS ANGELES

SYMMETRIC FUNCTIONS OF n-IC RESIDUES (mod p)†

BY T. A. PIERCE

If p be an odd prime, q is said to be an n-ic residue of p if the congruence $x^n \equiv q \pmod{p}$ has solutions; otherwise q is an n-ic non-residue of p. A necessary and sufficient condition that q be an n-ic residue of p is that

$$(1) \qquad q^{(p-1)/\delta} \equiv 1, \quad (\mathrm{mod}\ p),$$

where $\delta = \text{g.c.d.}\ (p-1, n)$. The number‡ of n-ic residues of a given prime p is $(p-1)/\delta$.

It is with the symmetric functions of these n-ic residues that this paper deals.

By means of (1) we readily prove that the product of two n-ic residues is an n-ic residue and that the product of an n-ic residue by an n-ic non-residue is an n-ic non-residue.

Put $(p-1)/\delta = r$ and let $q_1, q_2, \cdots, q_r$ be the set of all distinct n-ic residues of p. Then $q_iq_1, q_iq_2, \cdots, q_iq_r$ is the same set in different order, for the assumption that two members of this last set are congruent leads to the conclusion that two members of the first set are not distinct.

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* Loc. cit., p. 140.

† Presented to the Society, March 30, 1929.

‡ Dirichlet-Dedekind, *Zahlentheorie*, 4th ed., 1894, p. 74.