NON-EXISTENCE THEOREMS ON THE NUMBER OF REPRESENTATIONS OF ARBITRARY ODD INTEGERS AS SUMS OF 4r SQUARES*

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1. *Introduction.* The theorems stated in §3 and proved in §4 will be more significant if we first outline some known results and devise a definition which they suggest. In §5 an interesting problem is proposed, to which the method of this paper is at least partly applicable.

2. *Simplicity of the Number of Representations of an Integer as a Sum of* 4r *Squares.* Let n, r be given integers. The number $N(n, r)$ of one-rowed matrices $(x_1, x_2, \cdots, x_r)$ of integers $x_j \lesseqgtr 0$ $(j=1, 2, \cdots, r)$ such that $x_1^2+x_2^2+\cdots+x_r^2=n$ is called, as customary, the number of representations of n as a sum of r squares; $N(0, r)=1$. Henceforth let n be an arbitrary integer >0, and m an odd integer >0. Denote by $\zeta_j(n)$, $j\geqq 0$, the sum of the jth powers of all the divisors of n, $\zeta_j(0)=1$ by convention; and by $\xi_j(n)$ the sum of the jth powers of all the divisors $\equiv 1 \bmod 4$ of n minus the like sum for the divisors $\equiv 3 \bmod 4$. Write $(-1\,|\,m) \equiv (-1)^{(m-1)/2}$. Then, either from the analysis of Bulyguin† or otherwise, it is known that the general structure of $N(m, 2r)$ is as follows:

$$N(m,4r) = a\zeta_{2r-1}(m) + F_r(m),$$

$$N(m,4r-2) = [b + c(-1\,|\,m)]\xi_{2r-2}(m) + G_r(m),$$

where a, b, c are numerical constants (independent of m) different from zero; $F_j(m)=G_j(m)=0$ $(j=1, 2)$, and $F_r(m)$, $G_r(m)$, when $r>2$, are sums of homogeneous polynomials in the integers $y_1, y_2, \cdots, y_{2t} \gtreqless 0$ such that

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