

TOWNSEND ON REAL VARIABLES

Functions of Real Variables. By E. J. Townsend. New York, Henry Holt, 1928. 405 pp. \$5.00.

This book, as indicated in the preface, is a text-book on the subject, and not a treatise. The material used has been divided into seven chapters of approximately equal length, except for the first. They are as follows: Chapter I, *Real Number System*,—a short discussion of real numbers as defined by Cantor and Dedekind; Chapter II, *Theory of Point Sets*,—the usual properties involved in the notions of open and closed sets and measure, chiefly confined to linear sets; Chapter III, *Continuity and Discontinuity of Functions*,—including double and iterated limits, continuity in one or two variables, semi-continuous and pointwise discontinuous functions; Chapter IV, *Derivatives and their Properties*,—derivatives, upper and lower derivatives, law of the mean, partial derivatives, and total differentiability; Chapter V, *Riemann Theory of Integration*,—the usual theory, including improper integrals and integrals involving a parameter; Chapter VI, *Lebesgue and Other Integrals*,—the usual definitions and theory, including primitive functions and a short account of the work of Stieltjes' Hellinger, Denjoy, and Perron; Chapter VII, *Infinite Series*,—uniform and quasi-uniform convergence of series of functions, with applications to differentiation and integration, properties of power series (not including Taylor's series), condensation of singularities, and various methods of handling divergent series. Mention should also be made of the collections of problems at the ends of the chapters and the numerous references to original sources, both of which should be useful to teacher and student.

In criticizing this book it would be natural to compare it with the other two books on the same subject in English, those by Hobson and by Pierpont, but it must be remembered that they are essentially treatises, while the book under consideration is a comparatively short text-book and must be judged accordingly. A discussion of its merits falls naturally under three heads: its aesthetic qualities, the choice of material, and its utility.

With regard to the first the book is excellent. Not only is the author's style good, but he has a manner of presentation that makes the book intensely interesting. The publishers, also, are to be congratulated on their workmanship, as the arrangement and printing are artistic and there is a noticeable freedom from misprints.

The choice of material appeals to the writer in general, but not entirely so. The chief sin of omission is the confining of the point-set theory to linear sets and the restriction of so many of the theorems on functions to functions of one variable. To be sure, most of the generalization is immediate, but in some cases this is not true and the beginner cannot be expected to know when this is the case. It also seems strange to see no discussion of Taylor's series and the conditions for its validity in a chapter devoted to infinite series, especially when nearly eight pages are given to power series. On the other hand it is the writer's opinion that the author has to some extent