

IRREDUNDANT SETS OF POSTULATES FOR THE LOGIC OF PROPOSITIONS*

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1. *Introduction.* Alonzo Church† has introduced the interesting notion of “irredundance” in connection with a set of postulates. A set of postulates is irredundant “if the postulates are independent and no one of them can be weakened with respect to the set.” I give in this paper a number of irredundant sets of postulates for the logic of propositions.‡

Church gives a “mechanical” method by which any set of independent postulates can be made irredundant: if

(I) A, B, C

is a set of independent postulates, the set

(II) $A, \text{ if } A \text{ then } B, \text{ if } A \text{ and } B \text{ then } C$

is equivalent to (I) and is irredundant. My postulate sets below have the form (II). But they are free from the irrelevances that (II) usually has when obtained mechanically by Church’s rule. The postulates are simple, and in every case the hypothesis is necessary to the conclusion.

There should, of course, be no objection to an irredundant set of postulates simply because it is in form (II).§ For, as Church has shown, ¶ *any* irredundant postulate set is equivalent (postulate for postulate) to an irredundant set of form (II). In fact, it can easily be shown that if postulate

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† Alonzo Church, *On irredundant sets of postulates*, Transactions of this Society, vol. 27 (1925), p. 318.

‡ For the nature of the logic of propositions see B. A. Bernstein, *Sets of postulates for the logic of propositions*, Transactions of this Society, vol. 28 (1926), p. 472. I shall refer to this paper in succeeding footnotes as Paper I.

§ Compare H. M. Gehman, this Bulletin, vol. 32 (1926), p. 159.

¶ Alonzo Church, this Bulletin, vol. 32 (1926), p. 626.