THE INVERSE PROBLEM OF THE CALCULUS OF VARIATIONS IN A SPACE OF $(n+1)$ DIMENSIONS*

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1. *Introduction.* Among the various types of inverse problems of the calculus of variations are those of Darboux, Hamel, Hirsch, and Kürschak.† Darboux discussed the problem of the plane showing that for a given equation of the form $y''=\phi(x, y, y')$ there exist an infinity of functions $f(x, y, y')$ such that the integral $\int_{x_1}^{x_2} f(x, y, y')dx$ taken along one of the integral curves of the given equations furnishes a maximum or minimum. Hamel found the general type of integral whose minimizing arcs are straight lines. Of the last two Hirsch considers an equation of the type $F(x, y, y', y'', \cdots, y^{(n)})=0$ and Kürschak generalizes this by introducing n independent variables. In both of these cases it was found that a necessary and sufficient condition for a given equation of the type considered to give a solution of a problem in the calculus of variations is that it have its equation of variation self-adjoint. No such restriction was found in Darboux's problem; however, it is well known that every differential equation of the second order for plane curves may be transformed into one whose equation of variation is self-adjoint.

The inverse problem of the calculus of variations for three-dimensional space is treated in my thesis.‡ It is the

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† Darboux, *Théorie des Surfaces*, vol. 3, §606.

A. Hirsch, *Ueber eine Charakteristische Eigenshaft der Differentialgleichungen der Variationsrechnung*, Mathematische Annalen, vol. 49, p. 49.

J. Kürschak, *Ueber die Transformation der partiellen Differentialgleichungen der Variationsrechnung*, Mathematische Annalen, vol. 56, p. 155.

G. Hamel, *Geometrieen, in denen die Geraden die Kurzesten sind*, Mathematische Annalen, vol. 59, p. 255.

‡ *Inverse problem of the calculus of variations in higher space*, written under the direction of G. A. Bliss, University of Chicago, 1926. Published in the Transactions of this Society, vol. 30 (1928), pp. 710–736.