The proof is immediate, for by the Hamilton-Cayley theorem

$$\delta(R(x)) = 0, \qquad \delta'(S(x)) = 0.$$

Since $\mathfrak{A}$ is isomorphic with the algebra of matrices $R(x)$ (or $S(x)$), we have $\delta(x)=0$ (or $\delta'(x)=0$).

For the example of §4 we have

$$\delta(\omega) = \omega^2 - \omega x_1, \qquad \delta'(\omega) = \omega^2 - 2\omega x_1 + x_1^2 .$$

Hence $\delta(x)=0$, while $\delta'(x)=x_1^2-x_1^2e_1-x_1x_2e_2$.

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ON THE NUMBER $(10^{23}-1)/9$

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The purpose of this note is to save any further effort* in trying to factor the number $N=(10^{23}-1)/9=111,\ 11111,\ 11111,\ 11111,\ 11111$ which in a previous paper was found to be composite.† This assertion was based on a negative result giving $3^{N-1}\not\equiv 1 \pmod{N}$.

On the basis of this conclusion Kraitchik‡ attempted to factor N arriving at another negative result that N had no factors and therefore was a prime. This conflict of results led us to recompute the value of $3^{N-1} \pmod{N}$ which shows clearly a mistake in the original calculation arising from the choice of 3 for a base instead of another number prime to $10^{23}-1$. Such another base would have furnished the extra check which would have detected the error.

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* A recent letter from Mr. R. E. Powers informs us that he has been to the trouble of finding 150 quadratic residues of N.

† This Bulletin, vol. 33 (1927), p. 338.

‡ Mathesis, vol. 42 (1928), p. 386.