EISENHART'S COLLOQUIUM LECTURES

Non-Riemannian Geometry. By Luther Pfahler Eisenhart. New York, American Mathematical Society, 1927. viii+184 pp.

Generalization having become a familiar process to mathematicians, it is not strange that Riemannian geometry was hardly sixty-four years old before the invention of the non-Riemannian. The former had its origin in Riemann's Habilitationsschrift (1854); the latter, in an article by Weyl (Mathematische Zeitschrift, vol. 2, 1918). Just a decade after the creation of non-Riemannian geometry Professor Eisenhart has given in his colloquium lectures an account of the essential features of the new theory with special reference to the lines along which it has been developed in this country, notably at Princeton.

The fundamental idea of non-Riemannian geometry is the association with a given manifold of a connection which forms a basis for comparing vectors at different points. In the first chapter of Eisenhart's book the asymmetric type of connection is studied, and parallelism of vectors is discussed. The author is careful to distinguish between vectors arising by parallel displacement and parallel vectors: given a vector v at a point x and a curve C passing through x, there is at any point x' of C a unique vector v' which arises by parallel displacement of v along C, whereas all vectors $\phi \cdot v'$, where ϕ is an arbitrary factor and v' is the vector just defined, are called parallel to v with respect to C. The definition of parallel displacement, which is given only for a curve, might well have been stated in the following more general form: the vector v undergoes parallel displacement as x describes any integral variety of the differential equations

$$v^i{}_{,j}\, dx^j = 0,$$

where $v^i{}_{,j}$ denotes the covariant derivative of v. If the vector field is assigned a priori so that $|v^i{}_{,j}| \neq 0$, the integral variety referred to in the suggested modification of the definition is a point, and the vector actually undergoes no displacement. If the connection is euclidean and the coordinates cartesian, the integral variety is the whole of space for any vector field with constant components. From the notion of parallelism are obtained the equations of the paths of the space, a path being defined as a curve whose tangents are parallel with respect to the curve. The paths are thus the straightest lines of the space.

In §8 incompletely integrable systems of total differential equations are discussed. The author proves a theorem which is fundamental in the subsequent treatment (Chapters II and III) of the equivalence problems. The later repetition of this theorem adapted to the particular problem in hand becomes rather monotonous. It would perhaps be sufficient to set up the integrability conditions in each case and to make a concise reference to the original statement of the theorem.

The remainder of Chapter I is devoted to the parallel displacement of