GERMAN EDITION OF DICKSON ON ALGEBRAS

Algebren und ihre Zahlentheorie. By L. E. Dickson. *Mit einem Kapitel über Idealtheorie von A. Speiser.* Zürich und Leipzig, Orell Füssli Verlag, 1927. 308 pp.

Another notable advance in the development of the subject of linear algebras occurred with the appearance of this book. The mathematical world naturally expects a high standard in the case of treatises by Professor Dickson and the present one will not be found disappointing. While to a certain extent a revision and translation into the German language of his earlier book, *Algebras and their Arithmetics* (reviewed in this Bulletin, vol. 30 (1924), pp. 263–270, by Professor Olive C. Hazlett), it contains much material that is new and interesting, particularly along the lines of extension of the theory of division algebras and of the theory of numbers. Included in the latter category is the final chapter on ideal theory by Professor A. Speiser, under whose supervision the translation was made and who has written the preface.

On the basis of the new results contained in this book the first award of the Cole Prize by this Society was made at its meeting in Chicago in April, 1928. It is worth noting also that some years earlier the annual prize of the A. A. A. S. was awarded to Professor Dickson for a paper of unusual merit on the same subject.

It will be sufficient here to discuss the chief ways in which this book differs from the earlier one. Aside from a rearrangement of the material the first thing that strikes the reader's attention is the extension of the theory of linear associative algebras contained in sections 34–44. While based on an article by the author in volume 28 of the Transactions of this Society, it nevertheless differs considerably from that in that it avoids the use of the theory of finite groups. Whether this is really an advantage may perhaps be debatable.

The author deals here with a division algebra A over a general field K that contains a principal unit and that has the four properties: (I) A is of order n^2; (II) it contains an element i that satisfies an equation, $f(\omega)=0$, irreducible in K; (III) the only elements in A that are commutative with i are polynomials in i with coefficients in K; (IV) all the roots of $f(\omega)=0$ are rational functions of i with coefficients in K. In view of a theorem proved later, however, the only condition, as the author points out, that really restricts in any way the generality of the attack on the determination of associative division algebras is the last.

For the case where the irreducible equation $f(\omega)=0$ is "cyclic," to use the terminology of finite groups, algebras of this sort had been determined previously by the author and had been discussed in the English edition. The notable advance here consists in the use of non-cyclic equations. The chief difficulty of the problem lies in the imposition of the associative law, which leads to some conditions that are not particularly simple. Any doubt