AN APPLICATION OF TENSOR ANALYSIS TO THE FIRST VARIATION OF AN INTEGRAL*

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The method which J. L. Synge† has used to develop the first and second variations of an integral of a restricted type is readily applicable so far as the first variation is concerned, to a general integral of a *regular* calculus of variations problem. Let C be a curve in an n-dimensional space

$$(1)\qquad x^\alpha = x^\alpha(u), \qquad (u_1 \leqq u \leqq u_2), \qquad (\alpha = 1,2, \cdots, n).$$

The value of the integral

$$(2)\qquad t = \int_{u_1}^{u_2} F(x,\dot{x})du$$

will be called the *arc length* of the curve.‡ Let the curve C be imbedded in a one-parameter family of comparison curves

$$(3)\qquad x^\alpha = x^\alpha(u,v), \qquad (u_1 \leqq u \leqq u_2, v_1 \leqq v \leqq v_2),$$

such that C is given by $v=0$, that is, $x^\alpha(u, 0)=x^\alpha(u)$ identically in u. We suppose the limits of integration to be unvaried and so the value of the integral t taken along any one of the curves $v=$constant is a function of v alone. The equations (3) define a surface in terms of the parameters u and v. Let the tangent vectors to the u and v-curves be designated by ξ and η respectively,

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† *The first and second variations of the length-integral in Riemannian space*, Proceedings of the London Mathematical Society, vol. 25 (1925), pp. 247–264.

‡ For the conditions imposed on the function F and the notation here employed see the author's paper, *Parallelism and transversality in a subspace of a general (Finsler) space*, Annals of Mathematics, vol. 28 (1927), pp. 620–628.