$$\frac{m^3(1 - m^3)^3}{(1 - 20m^3 - 8m^6)^2} = \frac{-\left(\dfrac{1+2m^3}{6m^2}\right)^3\left[1+\left(\dfrac{1+2m^3}{6m^2}\right)^3\right]^3}{\left[1+20\left(\dfrac{1+2m^3}{6m^2}\right)^3-8\left(\dfrac{1+2m^3}{6m^2}\right)^6\right]^2}.$$

Dividing out the factors $(1-20m^3-8m^6)^2(8m^3+1)^3$, to which correspond special cubics, we obtain an equation of degree 24. The 24 cubics corresponding are exactly the 24 previously mentioned and we have proved that *a necessary and sufficient condition that a non-special cubic have its* (*Hessian*)$^3$ *coincide with itself, is that its Hessian be projectively equivalent to itself.*

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# A PROOF OF THE FUNDAMENTAL THEOREM OF ALGEBRA*

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1. *Introduction.* The number of proofs given of the fundamental theorem of algebra is large. Perhaps for that very reason still another proof may not be unacceptable. The one that is offered here is not "elementary," since it makes use of some general results in analysis. Yet it may be termed simple, and may be of interest. It is our hope that this proof, which is believed to be new, may, with no great embarrassment, take its place in the family of proofs that every algebraic equation has a root.

2. *The Proof.* We consider the equation

$$(1)\quad a_kx^k + a_{k-1}x^{k-1} + \cdots + a_1x + a_0 = 0, \quad (a_k \neq 0, k > 0).$$

There is no loss in generality in supposing that‡ $a_1 \neq 0$; and,

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‡ For suppose $a_1=0$. Make the substitution $x=y+\alpha$. We obtain an equation in $y$, in which the coefficient of $y$ is a polynomial in $\alpha$ of degree