# CUBICS WHOSE (HESSIAN)$^n$'S ARE THEMSELVES*

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The four equianharmonic, four degenerate and six harmonic cubics which belong to the syzygetic pencil $\lambda C+\mu H=0$, where $C=0$ is a general cubic and $H=0$ is its Hessian, have been studied in much detail. We denote them as the 14 *special* cubics of the pencil. It is known that the Hessian of a degenerate or equianharmonic cubic is a degenerate cubic, and that the (Hessian)$^2$, that is, the Hessian of the Hessian—of a harmonic cubic is itself and that the harmonic cubics are characterized by this property. The more general problem has been considered by Hostinský only, who entirely by calculation has obtained certain results for $n=3$ and $n=4$.† In this note a characterizing property for $n=3$ is obtained and certain general results noted, the method being quite different. The theorems of this note are believed to be new, though a part of Theorem 2 could be obtained from the work of Hostinský.

Denote by $\gamma_n$ a cubic whose (Hessian)$^n$ is itself and whose (Hessian)$^t$ for every value $t<n$ is not itself. Let $m$ denote the parameter of a cubic, and $m^{(n)}$ the parameter of its (Hessian)$^n$; then

$$m^{(r+1)} = -\frac{1+2(m^{(r)})^3}{6(m^{(r)})^2}, \qquad (r=0,1,\cdots,n-1),$$

where $m^{(0)}=m$. Then $m^{(n)}=m$ gives an equation in $m$ of degree $3^n$, which is also satisfied by any cubic whose (Hessian)$^r$ is itself, for $r$ a divisor of $n$. Dividing out from the equation the factors corresponding to these cubics, one will have in general an equation whose roots give cubics $\gamma$.

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† B. Hostinský, *Sur les Hessiennes successives d'une courbe du troisième degré*, Proceedings of the International Congress, Cambridge, 1912, vol. 2, pp. 102–104.