GRASSMANN'S PROJECTIVE GEOMETRY, VOLUME II

Projektive Geometrie der Ebene unter Benutzung der Punktrechnung. Volume II: *Ternäres.* Part 1. By Hermann Grassmann. Leipzig und Berlin, B. G. Teubner, 1913. xii+410 pp.

Projektive Geometrie der Ebene unter Benutzung der Punktrechnung. Volume II: *Ternäres.* Part 2. By Hermann Grassmann. Leipzig und Berlin. B. G. Teubner, 1927. xvi+522 pp.

The first volume of this work was published in 1909 and reviewed in this Bulletin (1913) by L. W. Dowling. The first part of the second volume appeared in 1913, but, owing to the world war, the volume was not completed before December 1921, when the finished second part was deposited as manuscript in the author's desk. Hermann Grassmann, Jr. died the following month, Jan. 21, 1922. One of his students, G. Wolff in Hannover, took charge of the publication of this second part, but owing to financial difficulties the work did not appear before 1927. Among those giving financial aid we note our own E. Carus.

The first part, which has four chapters (Hauptteile), introduces us to the ternary field of projective geometry. A point x in the plane is given by the equation $x=\xi_1e_1+\xi_2e_2+\xi_3e_3$, where e_1, e_2, e_3 are the vertices of the fundamental triangle; ξ_1, ξ_2, ξ_3, the "Ableitszahlen," may be considered as the Grassmann coordinates of the point x. The dual of the point is the "stab," or directed line-segment which is bound to the line on which it lies, in the sense that it can only be carried along the line in its own direction or in the opposite direction. Any stab U in the plane is represented by the equation $U=u_1E_1+u_2E_2+u_3E_3$, where u_1, u_2, u_3 are the Grassmann line-coordinates and E_1, E_2, E_3 are "stabe" on the lines joining the fundamental points, or "grundstàbe," defined by the equations $E_1=[e_2e_3]$, $E_2=[e_3e_1]$, $E_3=[e_1e_2]$. The unit point e, mass m, is now defined as a point with coordinates (1, 1, 1), so that $e=e_1+e_2+e_3$; the masses m_1, m_2, m_3 may be so chosen that the exterior three-point product $[e_1e_2e_3]=1$ (blatteinheit). Dually we have a unit "stab" of length l such that $E=E_1+E_2+E_3$ and $[E_1E_2E_3]=[e_1e_2e_3]=1$. It is then shown that the ratios $\xi_1:\xi_2:\xi_3$ and $u_1:u_2:u_3$ may be geometrically interpreted as double ratios

$$\frac{\xi_2}{\xi_3}=\frac{p_2}{p_2'}:\frac{p_3}{p_3'},\qquad \frac{\xi_3}{\xi_1}=\frac{p_3}{p_3'}:\frac{p_1}{p_1'},\qquad \frac{\xi_1}{\xi_2}=\frac{p_1}{p_1'}:\frac{p_2}{p_2'},$$

where p_1, p_2, p_3 are the distances of the point x from the sides of the fundamental triangle, and p_1', p_2', p_3' those of the unit point e from the same sides respectively. Dually we obtain for the ratios $u_1:u_2:u_3$ the values

$$\frac{u_2}{u_3}=\frac{q_2}{q_2'}:\frac{q_3}{q_3'},\qquad \frac{u_3}{u_1}=\frac{q_3}{q_3'}:\frac{q_1}{q_1'},\qquad \frac{u_1}{u_2}=\frac{q_1}{q_1'}:\frac{q_2}{q_2'},$$

q_1, q_2, q_3 being the lengths of the perpendiculars from the vertices e_1, e_2, e_3 on the stab U, and q_1', q_2', q_3' those of the perpendiculars from the same