

ON THE REMOVAL OF FOUR TERMS FROM AN  
EQUATION BY MEANS OF A TSCHIRNHAUS  
TRANSFORMATION\*

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The removal of the second, third, and fourth terms from an equation  $f(x) = 0$  by means of a Tschirnhaus transformation is a matter which can be summarized in two statements. First, if  $f(x) = 0$  is of higher than the fourth degree, the well known Jerrard process, which uses a fourth degree transformation, serves to remove the three terms without requiring the solution of any equation of degree greater than three. Second, if  $f(x) = 0$  is of the fourth degree, Lagrange first showed that a similar reduction is possible, employing a third-degree transformation.† Lagrange's transformation does not, however, apply to higher degree equations; the fact that the transformed quartic is a binomial equation is essential to its success.

The removal of the second, third, fourth, and fifth terms leads to a larger number of different cases. In general, we can say that, since the equations of condition are of degree 1, 2, 3, 4, we may be led to an equation of the 24th degree. But this maximum can almost always be substantially reduced. Thus Hamilton, in his report on the validity of Jerrard's work, was able to show that, if  $f(x) = 0$  is of degree greater than ten, the transformation can be effected with the aid of a single quartic equation, and no equation of higher degree, while simply one additional quartic has to be introduced if  $f(x) = 0$  is of degree ten.‡ Sylvester later obtained

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† See his *Oeuvres*, vol. III, Paris, 1869, pp. 284-295. The article, his well known *Réflexions sur la résolution algébrique des équations*, was first published in 1770-1771.

‡ Sixth Report of the British Association for the Advancement of Science, London, 1837, pp. 295-348. See, in particular, p. 318 ff.