CONCERNING COVARIANTS OF THE RATIONAL PLANE QUARTIC CURVE WITH COMPOUND SINGULARITIES*

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1. *Introduction.* After investigating the invariants of the rational plane quartic curve R_2^4 with compound singularities† it is of interest to see what effect such singularities have on covariants of the curve. This paper observes the effect of the tacnode and the ramphoid cusp upon associated curves and sets of covariant parameters.

2. *Covariant Forms of the Curve with a Tacnode.* To examine these forms we let $t=\pm a$ be the tacnodal parameters and t_1 and t_2 the contacts of the distinct double line of the curve with an acnode. This may be done if 0 and ∞ are the contacts of the two tangents to the curve from the tacnode.‡ These tangents and the line through their contacts give the very neat representation

$$(1)\qquad \begin{cases} x_0 = t^4 - a^2t^2, \\ x_1 = t^2 - a^2, \\ x_2 = t^3 + st^2 + \sigma_2 t, \end{cases}$$

where $\sigma_1 = t_1+t_2$, $\sigma_2 = t_1t_2$, and $s = [a^2\sigma_1^2 + (a^2+\sigma_2)^2]/(2a^2\sigma_1)$.

The pencil of conics§

$$(2)\qquad g_2 - \lambda K = 0,$$

where g_2 is the envelope of lines which cut the curve in self-apolar sets of points and K is the locus of the vertices

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† Neelley, *Compound singularities of the rational plane quartic curve*, American Journal of Mathematics, vol. 49 (1927), pp. 389–400.

‡ Neelley, loc. cit., p. 394.

§ J. E. Rowe, *Covariants and invariants of the rational plane quartic*, Transactions of this Society, vol. 12 (1911), pp. 298–299.