THE THEORY OF MULTIPLE IMPLICATION AND ITS APPLICATION TO THE GENERALIZED PROBLEM OF EPIMENIDES

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Let us allow $p, q, r, \cdots$ to stand for propositional functions, that is, for variables whose truth-values are in general dependent on the meaning of the terms that enter into them, and let the expression $(p\ q\ r \ldots)$ have the following verbal interpretation:

"p, q, r, $\cdots$ are simultaneously true for some meanings of the terms that enter into them."

We shall speak of this expression, $(p\ q\ r \cdots)$, or its negative, as the *existential* of the *n elements*, p, q, r, $\cdots$, and we shall say that it is of the first *order* and *n*th *degree*. The function containing existentials of the first order or their negatives as elements, will be of the second order and so on. For the particular case of one variable we should write:

$(p) \;= p$ is true for some meanings of the terms,

$(p)' \;= p$ is true for no meanings of the terms,

$(p')' = p$ is true for all meanings of the terms,

$(p') \;= p$ is true for not all meanings of the terms,

the prime being used as the symbol of denial or negation.

The effect of the symbol ( ), regarded as an operator, will be in general to weaken the expression on which it operates, unless this expression be zero (o) or one (i). Thus,

$$p \text{ implies } (p),$$

$$(p')' \text{ implies } p,$$

but not conversely. This simple provision is important, for it will lead at once to the result that certain generally recognized principles are untrue in a logic of complete generality. Thus it will turn out that the equality,

$$p \text{ implies } q = p' \text{ or } q$$