CONCERNING R. L. MOORE'S AXIOMS Σ_1 FOR PLANE ANALYSIS SITUS*

BY R. L. WILDER

1. *Introduction.* R. L. Moore has proposed† a system, Σ_1, of eight axioms for plane analysis situs. That a space S satisfying this system is in one-to-one continuous correspondence with two-dimensional euclidean space was shown by Moore in a later paper.‡

It is the purpose of the present paper to show that the set Σ_1 may be reduced to a set of *seven* Axioms, by the elimination of Axiom 6, which is a consequence of the other Axioms. Doubt as to the independence of Axiom 6 was raised in the mind of the author by noticing that the independence examples given for Axioms 6 and 7 on pp. 162 and 163 of F.A. are not valid, and by the subsequent finding of an independence example for Axiom 7 accompanied by failure to find any independence example for Axiom 6.

2. *Independence of Axiom 7.* The independence of Axiom 7 is established be the following example:

Let the space considered by ordinary euclidean space of two dimensions. Choose a pair of rectangular axes OX and OY. For every positive integer n, let points be defined as follows: $A_n=(0,-1/n)$, $B_n=(1/n,-1/n)$, $C_n=(1/n,\ 1/n)$, $D_n=(0,\ 1/n)$, $E_n=(0,\ 0)$, $F_n=(1/(2n),\ 0)$. Let T_n be the bounded domain whose boundary is the rectangle $A_nB_nC_nD_n$ together with the straight line interval E_nF_n. Then a point set R is a region if and only if R is either the interior of some simple closed curve or identical with some T_n. That the

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† *On the foundations of plane analysis situs*, Transactions of this Society, vol. 17 (1916), pp. 131–164. Referred to hereafter as F. A.

‡ *Concerning a set of postulates for plane analysis situs*, Transactions of this Society, vol. 20 (1919), pp. 169–178.