ON THE LOCI OF THE LINES INCIDENT WITH k $(r-2)$-SPACES IN S_r

BY B. C. WONG

The problem of the determination of the number N_r of lines that meet $2r-2$ given $(r-2)$-spaces in S_r has been solved.* Schubert's symbolic or enumerative method is powerful for the solution of problems of this kind and has indeed been the one used, but it does not offer any insight into the nature of the geometry involved. It is the purpose of this paper to re-determine the number N_r and also to obtain the loci of the ∞^{2r-2-k} lines incident with k given $(r-2)$-spaces in S_r, where $r<k\leqq 2r-2$.

For our purpose we make use of the known theorem:† *The locus of the ∞^{r-2} lines incident with r general $(r-2)$-spaces in S_r is a hypersurface V_{r-1}^{r-1}.*

Now consider r of the given $(r-2)$-spaces, say $S_{r-2}^{(i)}$ $[i=1, 2, \cdots, r]$. They yield a V_{r-1}^{r-1} whose generators are incident with them. Any of the remaining $k-r$ given $(r-2)$-spaces, say $S_{r-2}^{(r+1)}$, meets V_{r-1}^{r-1} in a V_{r-3}^{r-1}. Any hyperplane S'_{r-1} through $S_{r-2}^{(r+1)}$ meets $S_{r-2}^{(i)}$ in r $(r-3)$-spaces. The ∞^{r-4} lines that meet these r $(r-3)$-spaces are in S'_{r-1} and hence meet $S_{r-2}^{(r+1)}$, and they form a $V_{r-3}^{M_r'}$ whose order M_r' is to be determined later. Hence the ∞^{r-3} lines incident with $r+1$ $(r-2)$-spaces in S_r form a $V_{r-2}^{M_r+M_r'}$ (writing M_r for $r-1$), for it is met by S'_{r-1} in a $V_{r-3}^{M_r+M_r'}\equiv V_{r-3}^{M_r}+V_{r-3}^{M_r'}$.

Now the $(r+2)$th $(r-2)$-space, $S_{r-2}^{(r+2)}$, meets $V_{r-3}^{M_r+M_r'}$ in a $V_{r-4}^{M_r+M_r'}$. A hyperplane S''_{r-1} through $S_{r-2}^{(r+2)}$ intersects the other $r+1$ $(r-2)$-spaces in $r+1$ $(r-3)$-spaces and the ∞^{r-5} lines incident with the latter form a $V_{r-4}^{M_r''}$. Hence the

* See C. Segre, *Mehrdimensionale Räume*, Encyklopädie der Mathematischen Wissenschaften, vol. III: 2, pp. 813, 814 where full references are given.

† This Bulletin, vol. 32, No. 5, pp. 553–554.