SCHMIDT'S ORTHOGONAL ENNUPLE AND THE FRENET FORMULAS FOR A CURVE

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This note deals with §32 (pp. 103–107) of Eisenhart's *Riemannian Geometry*.

The object of the above section is to define an orthogonal ennuple from a given set of n linearly independent vectors and hence to deduce the Frenet formulas of a curve in a space of n dimensions. One criticism of Eisenhart's ennuple is that when the original set of vectors forms an orthogonal ennuple the defined ennuple does not reduce to it, and in this note I have modified the definition so that this disadvantage is removed. Also, to obtain equations (32.18) in his book, Eisenhart has assumed that $b_{p-1} = (b_{p-1}^2)^{1/2}$, and this is incorrect if we understand that the positive root has been taken.

Being given a set of n linearly independent vectors,

$$\xi_{\alpha|}^{r} \qquad (\alpha = 1, 2, 3, \cdots, n), \tag{1}$$

in a Riemannian manifold of n dimensions, our object is to define from these vectors a set of n orthogonal unit vectors,

$$\lambda_{\alpha|}^{r}, \qquad (\alpha = 1, 2, 3, \cdots, n). \tag{2}$$

Building up the ennuple in the usual manner,* we arrive at the formulas

$$\lambda_{p|}^{r} = \epsilon_p(|\, b_p/b_{p-1} \,|)^{1/2} \sum_{\alpha=1}^{p} \xi_{\alpha|}^{r} B_p^{\alpha}, \quad (p = 1, 2, \cdots, n). \tag{3}$$

The quantities ϵ_p determine the orientation of the ennuple in space. Eisenhart takes all the ϵ's equal to $+1$. I propose to choose them so that when (1) is an orthogonal ennuple the set (2) may reduce to it. We easily find that we must

* See Kowalewski, *Determinantentheorie*, pp. 423–426.