

AN UNNECESSARY CONDITION IN TWO
THEOREMS OF ANALYSIS SITUS*

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1. *Introduction.* In this paper K will be a closed point set such that, if S refers to the space under consideration, $S - K = S_1 + S_2$. One or more of the following conditions will hold.

CONDITION 1. *Every simple continuous arc which joins a point of S_1 to a point of S_2 contains at least one point of K .*

CONDITION 2. *Every point P of S_i ($i=1$, and 2) can be joined to any point Q of K by a simple continuous arc PQ such that $PQ - Q$ is contained in S_i .*

CONDITION 3. *Every two points of S_i ($i=1$, and 2) can be joined by a simple continuous arc contained in S_i .*

Schoenflies,† using the definition of a simple closed curve given by Jordan, stated and proved the following theorem for a plane S .

THEOREM A. *If K is bounded and if Conditions 1, 2, and 3 hold, then K is a simple closed curve.*

This theorem was later proved by Lennes‡, using his own definition of a simple closed curve. In his paper *On the foundations of plane analysis situs*, R. L. Moore§ points out that

* Presented to the Society, December 29, 1925.

† A. Schoenflies, *Ueber einen grundlegenden Satz der Analysis Situs*, Göttinger Nachrichten, 1902, p. 185.

‡ N. J. Lennes, *Curves in non-metrical analysis situs with an application in the calculus of variations*, American Journal, vol. 33 (1911), pp. 287-326, Lennes defines a *simple continuous arc* from A to B as a closed, bounded, connected point set containing $A + B$ but containing no proper connected subset that contains $A + B$. He defines a *simple closed curve* as the sum of two simple continuous arcs which have their end points and only their end points in common.

§ Transactions of this Society, vol. 17 (1916), p. 131-164, Theorem 48. This paper will be referred to hereafter as F. A.