$$(9)\qquad \begin{vmatrix} A & B & C \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} = 0, \quad \begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix} \cdot \begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix} = \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}^2 \cdot O$$

represent the two eliminants of the system (8).

The two necessary and sufficient conditions to be fulfilled by three arbitrary functions $H(x, y)$, $K(x, y)$, and $R(x, y)$ of x and y in order that the congruence of circles

$$(\alpha - H)^2 + (\beta - K)^2 = R^2$$

which they determine in the (α, β)-plane may represent the derivative of a polygenic function are obtained by retransforming the two equations (9) *from the quantities u, v, M, N, O into the quantities x, y, H, K, R; the two final conditions contain R and the derivatives of H, K and R up to the third order.*

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ON THE INVERSION OF ANALYTIC TRANSFORMATIONS*

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We wish to consider the transformation

$$(1)\qquad x_i = f_i(y_1, \cdots, y_n), \qquad (i = 1, \cdots, n),$$

in the neighborhood of the origin $(y) = (0)$, at which point the functions f_i are analytic, and vanish simultaneously. We are interested in the case in which the jacobian

$$J = \frac{\partial(f_1, \cdots, f_n)}{\partial(y_1, \cdots, y_n)}$$

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The methods and point of view of the second chapter of W. F. Osgood's *Lehrbuch der Funktionentheorie* are assumed throughout this paper. Our results may be regarded as the extension of §20 of that work.

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