NOTE ON THE DERIVATIVE CIRCULAR CONGRUENCE OF A POLYGENIC FUNCTION*

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In the first paper† I published on polygenic functions I stated the following theorem.

If

$$w = \phi(x, y) + i\psi(x, y)$$

is a polygenic function of

$$z = x + iy$$

then the first derivative of w with regard to z, $dw/dz=\gamma=\alpha+i\beta$, is represented in the γ-plane by the congruence of circles

$$(1) \qquad (\alpha - H)^2 + (\beta - K)^2 = R^2$$

where

$$(2) \qquad \begin{cases} 2H = \phi_x + \psi_y, \quad 2K = -\phi_y + \psi_x, \\ R^2 = h^2 + k^2, \\ 2h = \phi_x - \psi_y, \quad 2k = \phi_y + \psi_x. \end{cases}$$

To every point (x, y) of the z-plane corresponds by means of dw/dz that circle of (1) determined by the particular pair of values (x, y).

In this paper I wish to study the converse problem. Let $H(x, y)$, $K(x, y)$, and $R(x, y)$ be three arbitrary functions of (x, y)‡ and for every pair of values (x, y) construct in the γ-plane the circle whose center is (H, K)

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† *A new theory of polygenic functions*, Science, vol. 66 (Dec., 1927), pp. 581–582. See also a previous paper by Hedrick, Ingold, and Westfall, Journal de Mathématiques, *Theory of non-analytic functions of a complex variable*, (6), vol. 2 (1923), pp. 327–342.

‡ We require, however, that these functions be continuous and have continuous first, second, and third derivatives.