NEW DIVISION ALGEBRAS

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1. *Introduction.* No technical acquaintance with linear algebras is presupposed in this note. We consider only linear algebras for which multiplication is associative. As with quaternions, an algebra A is called a *division* algebra if every element $\neq 0$ of A has an inverse in A. A division algebra A over a field F is called *normal* if the numbers of F are the only elements of A which are commutative with every element of A.

In a paper recently offered to the Transactions of this Society, A. A. Albert determined all normal division algebras of order 16 and found a new type. The object of this note is to derive from mild assumptions the corresponding type of normal division algebras A of order $4p^2$, where p is a prime. We shall first draw simple conclusions from an initial assumption.*

Assumption 1. Let A contain an element i_1 satisfying an equation $f(\omega^2)=0$ of degree $2p$ with only even powers of ω, whose coefficients are in F, that of ω^{2p} being unity, and which is irreducible in F, such that the polynomials in i_1 are the only elements of A which are commutative with every element of A.

2. LEMMA 1. *Let an element i_2 of A be commutative with $I=i_1^2$, but not with i_1 itself. The algebra S generated by i_1 and i_2 is of order $4p$. It may be regarded as an algebra of order 4 with the basis $1, i_1, i_2, i_1i_2$ over $F(I)$; this algebra is normal. In other words, the polynomials in I are the only elements of S which are commutative with every element of S.*

Let K denote the field composed of all those elements of

* Except for the requirement concerning even powers of ω, Assumption 1 is proved in the writer's *Algebren und ihre Zahlentheorie*, Zürich, 1927, pp. 262–3.