ON A CERTAIN SYSTEM OF ∞^{r-2} LINES IN r-SPACE

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This paper deals with the following theorem.

The locus of ∞^{r-2} lines incident with r given $(r-2)$-spaces in S_r is an $(r-1)$-dimensional manifold V_{r-1}^{r-1} of order $r-1$.

To prove this, we note that for $r=2$ there is one line joining two given points in a plane and that for $r=3$ the lines meeting three given lines in an S_3 form a quadric surface. If $r=4$, that is, if four planes are given in S_4, the locus of the ∞^2 lines incident with them is a V_3^3. This cubic hypersurface with its many interesting properties has been studied by a number of writers.*

If $r=5$, that is, if five three-spaces are given in S_5, pass an S_4 through one of them, say R_3. This S_4 meets the other four 3-spaces in four planes and the lines incident with these four planes are also incident with R_3. These lines form a V_3^3 in S_4. The manifold of the ∞^3 lines incident with the five given 3-spaces is intersected by the S_4 through R_3 in R_3 and a V_3^3 and is therefore of order 4. If we apply this process of reasoning to the cases $r=6$, 7, etc., we soon arrive at the general theorem stated above.

Consider another proof. Let the r given $(r-2)$-spaces in S_r be S'_{r-2}, S''_{r-2}, $\cdots$, $S^{(r)}_{r-2}$, and further let a general line l be given. The points of l determine with S'_{r-2}, S''_{r-2}, $\cdots$, $S^{(r-1)}_{r-2}$ $r-1$ projective pencils of hyperplanes. The rth $(r-2)$-space, $S^{(r)}_{r-2}$, intersects these pencils in $r-1$ pencils of $(r-3)$-spaces. As there are $r-1$ sets of corresponding $(r-3)$-spaces each intersecting in a point, there are $r-1$ lines of intersection of corresponding hyperplanes of the pencils in S_r which meet $S^{(r)}_{r-2}$. Hence, the general line l meets $r-1$ of the ∞^{r-2} lines

* See Bertini, *Projecktive Geometrie Mehrdimensionaler Räume*, 1924, Chap. 8, §§25–36, where references are given.