EVANS ON LOGARITHMIC POTENTIAL

The Logarithmic Potential, Discontinuous Dirichlet and Neumann Problems. By Griffith Conrad Evans. Colloquium Publications, American Mathematical Society, volume 6, 1927. viii+150 pp.

The appearance of this volume marks a broadening of the scope of the Colloquium Publications, in that these will no longer be confined to lectures given from time to time, by invitation of the American Mathematical Society, at its summer meetings. The new departure, however, could hardly have been initiated more smoothly or appropriately. For in the first place, Professor Evans has in the past been one of the Society's Colloquium lecturers, and secondly, his book is very closely related in character to the traditional Colloquium publications in that it is pre-eminently concerned with recent progress.

The book is characterized by the role played by Stieltjes and Lebesgue integrals and by functions of limited variation, and by the degree to which these concepts have enabled the author to express the conditions for his theorems in necessary and sufficient form. That these tools are particularly adapted to physical problems is a fact which the author early recognized. The material centers about a study of Poisson's integral in two dimensions and of the corresponding Stieltjes integral. The results are then extended to the integrals in terms of Green's functions for general regions.

In the introductory chapter, the fundamental properties of functions of limited variation and of Stieltjes integrals are succinctly developed, and some properties of Lebesgue integrals and theorems on the termwise integration of sequences are stated, with references or proofs. The last eight pages are given to Fourier series.

The second chapter develops properties of the Poisson-Stieltjes integral

$$\text{(A)} \qquad u(r,\theta)=\frac{1}{2\pi}\int_0^{2\pi}\frac{1-r^2}{1+r^2-2r\cos(\phi-\theta)}dF(\phi),$$

where $F(\phi)$ is of limited variation, and of the Poisson integral

$$\text{(B)} \qquad u(r,\theta)=\frac{1}{2\pi}\int_0^{2\theta}\frac{1-r^2}{1+r^2-2r\cos(\phi-\theta)}f(\phi)d\phi,$$

where $f(\phi)$ is summable.

The function $u(r,\theta)$ defined by (A) is harmonic $(r<1)$; the function

$$F(r,\theta)=\int_0^\theta u(r,\theta)d\theta$$

approaches, as $r\longrightarrow 1$, θ being fixed, the value

$$\tfrac{1}{2}\{F(\theta+0)+F(\theta-0)\}-\tfrac{1}{2}\{F(0+)+F(0-)\};$$

$u(r,\theta)$ is the difference of two non-negative functions for $r<1$; $u(r,\theta)$ is a function of θ of limited variation, uniformly as to r; at every point $P(1,\phi)$ at which $F(\phi)$ has a derivative $F'(\phi)=f$ (i.e., almost everywhere), $u(r,\theta)\to f$