HOMOGRAPHIC CIRCLES OR CLOCKS*

BY LULU HOFMANN AND EDWARD KASNER

1. *Introduction*. This paper contains some theorems on the homographic transformation of one circle into another. The distribution of points on the transformed circle is studied under the assumption that the distribution on the original circle is uniform. The varying *density* on the transformed circle is characterised *im Grossen* by the *centroid*, a point defined analytically by a mean-value process. At an individual point it is measured by the absolute value of the ratio of corresponding arc elements on the two circles, the ratio taken from the original to the transformed element.

The distribution of points on the transformed circle proves to be uniform when and only when the centroid coincides with the center. When this coincidence does not occur, the density varies as follows.

It reaches its maximum and its minimum, two reciprocal values, in the end points of the diameter through the centroid, the *main diameter*.

It assumes every intermediate value twice and to every value occurring the reciprocal value likewise occurs.

Points of equal density lie symmetrically with regard to the main diameter, and points of reciprocal densities are collinear with the centroid.

If we draw a chord through the centroid and number the end points and the corresponding segments determined by the centroid arbitrarily as the first and second, then the density in the first point will be equal to the ratio of the length of the second segment to that of the first segment.

2. *Homographic Clocks. The Centroid.* We shall treat the problem analytically. As the most convenient tool we choose the linear fractional transformation with complex coefficients of one complex variable into another

*Presented to the Society, February 6, 1928.