A THEOREM ON RULED SURFACES

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On page 229 of volume 1 of Fubini-Čech, *Geometria Proiettiva Differenziale*, appears the following theorem.

If the two flecnode curves of a ruled surface are plane, then the complex curves and the harmonic curves are plane, and all of their planes belong to a pencil.

The authors furnish a proof of this, and they call it "an interesting theorem by Sullivan."

In a letter to the author of this note, Professor C. T. Sullivan says, with reference to the above, "The theorem you refer to is consequently not to be found in any of my publications." It appears probable that the Italian geometers have confused this theorem with one of a somewhat similar nature published by the author in 1915,* or perhaps with a somewhat more general theorem contained in a later paper by the author.†

In this paper it is shown that the planes osculating the two branches of the flecnode curve and the planes osculating the two branches of the complex curve at the four points in which these curves cut a line element of their supporting ruled surface, will form a pencil, if and only if,

$$D \equiv p_{21}^{2}\Delta_1 - p_{12}^{2}\Delta_2 = 0, \tag{1}$$

where

$$\Delta_1 = p_{12}^{2}q_{22} - p_{12}q_{12}' + 3q_{12}^{2}, \quad \Delta_2 \equiv p_{21}^{2}q_{11} - p_{21}q_{21}' + 3q_{21}^{2},$$

p_{ik}, q_{ik} being coefficients of the flecnode canonical form of the system of differential equations defining the surface. The theorem of 1915 is a special case of that of 1923 since

*Transactions of this Society, vol. 16 (1915), p. 520. This volume also contains a paper by Professor Sullivan.

†Tôhoku Mathematical Journal, vol. 23 (1923), p. 114.