THE NODES OF THE RATIONAL PLANE QUARTIC*

BY L. T. MOORE

The projective properties of the rational quartic curve may be studied from two points of view. If the curve is given parametrically, its invariants are expressible in terms of the coefficients of the fundamental involution. If the curve is given by the plane $(KX)=0$, and the Steiner quartic surface $(X^{1/2})=0$, the invariants are expressible in symmetric functions of the coefficients of $(KX)=0$.† An invariant condition obtained for one form of the equations of the curve may readily be expressed in the invariants of the other form. The relations connecting the two systems are

$$(1)\qquad \begin{cases} 4I_2 = MS_1S_3, \\ I_2' = M(S_1S_3 - 16S_4), \\ I_4 = (16M)^2(S_2S_3^2 - S_1S_3S_4 + S_4^2), \\ I_6 = M^3S_3^2(S_1S_2S_3 - S_1^2S_4 - S_3^2), \end{cases}$$

where M is a positive constant. These equations may also be solved for any function of S_1, S_2, S_3, S_4, whose weight is a multiple of four.‡

To determine the nature of the nodes from the invariants, it is desirable to refer the quartic to a triangle whose vertices are at the nodes. Such a quartic is given by the equations§

$$X_0^2X_1^2 + X_1^2X_2^2 + X_0^2X_2^2 - 2X_0X_1X_2X_3 = 0,$$

$$\alpha_0X_0 + \alpha_1X_1 + \alpha_2X_2 + \alpha_3X_3 = 0.$$

Eliminating X_3 we have

$$(2)\qquad \alpha_3(X_0^2X_1^2 + X_1^2X_2^2 + X_0^2X_2^2) + 2X_0X_1X_2(\alpha_0X_0 + \alpha_1X_1 + \alpha_2X_2) = 0,$$

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† J. E. Rowe, Transactions of this Society, vol. 12 (1911), pp. 295–310.

‡ L. T. Moore, American Journal, vol. 48 (1926), p. 251.

§ The surface $(\sqrt{X})=0$ referred to a tetrahedron having the three double lines as edges. Salmon-Rogers, *Geometry of Three Dimensions*, vol. 2, p. 213.