

*Geometria Proiettiva Differenziale*. By G. Fubini and E. Čech. Bologna, Nicola Zanichelli, 1927. Vol. II, 406 pp.

The second volume of the Projective Differential Geometry of Fubini and Čech is a direct continuation of the first. Chapters, sections, and pages are numbered consecutively, the second volume beginning with Chapter 8, §71, page 389; and an index is provided at the end of the second volume for the entire work. Moreover, the continuity is not altogether mechanical. In the last chapter of the first volume it was proved that if a non-ruled surface in ordinary space is projectively deformable at all, then it is projectively deformable in  $\infty^3$ ,  $\infty^2$ , or  $\infty^1$  ways, and the concluding sections of that volume were devoted to a discussion of the first case. The opening chapter of the second volume completes the discussion of the other two cases.

Chapters 9 and 10 are devoted to a consideration of various portions of the projective differential geometry of a surface in the neighborhood of one of its points. Among the topics treated are the quadric of Moutard; Čech's transformation  $\Sigma$  between points in a tangent plane of a surface and planes through the contact point, with various special cases one of which is connected with a two-dimensional metric of Weyl; the canonical pencil of lines; and the cone of Segre. Arbitrary curvilinear coordinates are used in these chapters.

Complexes and congruences of lines are studied in Chapter 11. Here the closest possible analogy with the theory of surfaces is preserved. The theory proceeds from certain fundamental differential forms to the differential equations which determine the configuration except for a projective transformation. Much emphasis is placed on the notion of projective applicability of complexes and of congruences.

The projective differential geometry of hyperspace appears for the first time in Chapter 12. The now familiar transition is made from the fundamental differential forms for a hypersurface in  $S_n$  to the differential equations, which might themselves have been used as fundamental. Non-parabolic surfaces in  $S_4$  are studied, and ruled surfaces in spaces of even and odd dimensions are briefly considered.

It seems appropriate to remark in this connection that a general projective differential theory of a  $V_k$  in an  $S_n$ , with  $n > 4$  and  $1 < k < n - 1$ , remains yet to be constructed. Two methods seem to be available for constructing such a theory, namely, the method of Wilczynski who starts with a system of differential equations and employs the Lie theory of continuous groups, and the method of Fubini who starts with a system of differential forms and employs the absolute calculus of Ricci. But the labor involved in the former method seems at present to be prohibitive, and the latter method has so far failed because of the lack of a suitable fundamental quadratic form, or perhaps because of the lack of a suitable absolute calculus for an  $n$ -ary  $p$ -adic differential form.

Nearly the last third of the second volume is occupied by four appendices written by three different authors. In the first appendix Tzitzeica treats in French a special problem in the deformation of surfaces. In the second Bompiani gives a systematic exposition of his own contributions