

ON CONTINUOUS CURVES IN n DIMENSIONS*

BY G. T. WHYBURN AND W. L. AYRES†

If M_1 and M_2 are subsets of a connected point set M , the subset K of M is said to *separate* M_1 and M_2 in M if $M - K$ is the sum of two mutually separated sets containing M_1 and M_2 respectively. R. L. Moore‡ has shown that in order that a plane continuum M be a continuous curve§ it is necessary and sufficient that for every two distinct points A and B of M there should exist a subset of M which consists of a finite number of continua and which separates A and B in M . Consider the following example: Let S_i ($i = 1, 2$) be the set of all points (x, y, z) in three dimensions such that $x = (-1)^i$, $-1 \leq y \leq 1$, $0 \leq z \leq 1$. Let R_0 be the set of all points (x, y, z) such that $-1 \leq x \leq 1$, $-1 \leq y \leq 1$, $z = 0$. For each integer $n > 0$, let R_n be the set of all points (x, y, z) such that $-1 \leq x \leq 1$, $-1 \leq y \leq 1$, $z = 1/n$. Let

$$M = S_1 + S_2 + \sum_{n=0}^{\infty} R_n.$$

It is easy to see that every two points of M may be separated by a single subcontinuum of M and yet M is not a continuous curve. Hence the condition given by Moore is not sufficient in order that a continuum in n dimensions ($n > 2$) be a continuous curve. In this paper we give two modifications (Theorems 2 and 4) of Moore's theorem which hold in n dimensions.

* Presented to the Society, October 29, 1927.

† National Research Fellow in Mathematics.

‡ *A characterization of a continuous curve*, *Fundamenta Mathematicae*, vol. 7 (1925), pp. 302-307.

§ We shall use the term *continuous curve* in the sense of a point set which is closed, connected and connected im kleinen. See R. L. Moore, *Concerning simple continuous curves*, *Transactions of this Society*, vol. 21 (1920), p. 347.