

SHORTER NOTICES

Lehrbuch der Funktionentheorie. By L. Bieberbach. Band II: *Moderne Funktionentheorie.* Leipzig and Berlin, Teubner, 1927. vii+366 pp.

This work is the second volume of the set with the above title. Volume I on the elements of the theory of functions of a complex variable was reviewed by the present writer in this Bulletin (vol. 28 (1922), p. 467). The present volume treats of the more recent developments of the subject. The work is confined almost entirely to the functions of a single complex variable, with particular attention to certain special aspects. Functions of several variables in the complex domain are not treated. The author's own pet problem of conformal mapping, however, does not receive an unduly large proportion of space, though the scope of the entire book is not as complete as Bieberbach's article in the *Encyclopaedie der Mathematischen Wissenschaften*.

While Bieberbach in the main uses function theoretic methods without leaning very heavily upon real analysis, nevertheless, the use of very recent developments in the theory of functions of a real variable, starting with integrals of Lebesgue, have when combined with the older classical methods, given such elegant theorems as that of Fatou.

The eight chapters carry the following headings and indicate the topics treated. I. Conformal mapping. II. The elliptic modular functions. III. Bounded functions. IV. Uniformization. V. Picard's theorem. VI. Entire functions. VII. Analytic continuation. VIII. The Reimann zeta function.

In addition to the researches of the author, recent results of the following writers are presented: Carathéodory, Carlson, Fabry, Fatou, Hadamard, Hardy, Hoheisel, Harnack, Jensen, Jentzsch, Julia, Landau, Nevanlinna, Phragmen, Ostrowski, Schottky, Szegö, Vitali, Wigert, Wiman, and others.

Julia in his Borel monograph of 1923 gives Porter joint credit with Vitali for the important theorem to the effect that to every set of analytic functions uniformly bounded in a regular domain, there belongs an infinite subsequence having an analytic limit function. This theorem is closely related to the work of Osgood, Stieltjes, Schottky, and Montel. Similarly, Porter seems to have been the first to have pointed out that for any series with its circle of convergence as a natural boundary there exist certain "sub-series" which may be continued outside of the original circle of convergence. These are called "over-convergents" and ascribed by Bieberbach to Jentzsch. Finally, Bieberbach and others have failed to recognize that Porter in the *Annals of Mathematics* of 1907 assigned to his polynomial convergents of a given series, all the properties encompassed in Montel's normal families of functions.

The reviewer desires to bestow the very highest praise on the style of the present text. The author has a facile pen and achieves a happy result. The work is indispensable for any one interested in present day developments in analytic functions.

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