

THE PROBLEM OF DEPRECIATION IN THE CALCULUS OF VARIATIONS*

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1. *Introduction.* In a recent article Hotelling has shown that the older treatments of depreciation involve a number of serious errors of reasoning and has formulated the problem in such a way that many of these errors are overcome.‡ His view-point is that the owner of a machine will do everything in his power to maximize the present value of the sum of the anticipated rentals which the machine will yield from the present time t_1 to some future time t_2 plus the present value of the salvage value of the machine at the time t_2 when it is salvaged. Although the depreciation problem appears to be a Lagrange problem in the calculus of variations,§ Hotelling has chosen to consider it as a problem in the theory of ordinary maxima of functions.

If, in the light of recent developments in the new dynamical economics,|| the depreciation problem is formulated as a Lagrange problem with variable end-points, the resulting problem is sufficiently general to include as special cases all of the existing depreciation theories, i.e., such theories as the straight line, the compound interest, the sinking fund, the unit-cost-plus, and the theory due to Hotelling. In

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‡ H. Hotelling, *A general mathematical theory of depreciation*, Journal of the American Statistical Association, September, 1925.

§ Bliss, *Lectures on the Calculus of Variations*, University of Chicago, Summer, 1925, mimeographed by O. E. Brown, Northwestern University, Evanston, Ill.

|| Roos, *Dynamical economics*, Proceedings of the National Academy, vol. 13, No. 3 (March, 1927); Roos, *A dynamical theory of economics*, Journal of Political Economy, October, 1927. See also, Roos, *A mathematical theory of competition*, American Journal of Mathematics, vol. 57, No. 3, July, 1925, and G. C. Evans, *Dynamics of monopoly*, American Mathematical Monthly, vol. 31 (1924), No. 2.