EXPANSION IN SERIES OF NON-INVERTED FACTORIALS*

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An expansion of the form

$$f(z) = \sum \frac{b_n}{(z+1)(z+2)\cdots(z+n)}$$

can be obtained from the consideration of Cauchy's formula

$$2\pi i f(z) = \int_C \frac{f(t)dt}{z-t},$$

if $f(z) = 0$ at infinity, together with the result†

$$(1) \qquad \frac{n!}{(z+1)(z+2)\cdots(z+n+1)} = \int_0^1 u^n(1-u)^z du,$$

where $(1-u)^z$ denotes the branch reducing to unity for $u=0$. The above relations can also be used for deriving an expansion in series of non-inverted factorials. By (1) we have

$$\frac{1}{z-t} = \int_0^1 (1-u)^{z-t-1} du.$$

Consider

$$(1-u)^{z-t-1} = (1-u)^z(1-u)^{-t-1}.$$

Since

$$(1-u)^z = 1$$

when $u = 0$, we may write

$$(2) \quad (1-u)^z = 1 - \frac{z}{1!}u + \frac{z(z-1)}{2!}u^2 - \cdots + \frac{(-1)^n}{n!}z(z-1)\cdots(z-n+1)u^n + \cdots.$$

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† Whittaker and Watson, *Modern Analysis*, 3d edition, Cambridge University Press, 1920, p. 144.