SEPARATION THEOREMS AND THEIR RELATION TO RECENT DEVELOPMENTS IN ANALYSIS SITUS*

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1. *Introduction.* The study of those properties of point sets which are invariant under (1-1) continuous transformations form the subject matter of analysis situs. It is the purpose of the present paper to discuss one of the fundamental invariants of this field, that of a point set being separated by various of its subsets.

A set is said to be *connected in the weak sense*, if it is impossible to express it as the sum of two mutually separated sets, that is, two sets that are mutually exclusive and neither of which contains a limit point of the other one. A set M is said to be *connected in the strong sense* if, for every two points P and Q of M, there is in M a closed and connected subset containing both P and Q. If to the set K, consisting of $y=\sin (1/x)$, $[0<x\leqq 1/\pi]$, we add the origin, we obtain a set which is connected in the weak sense but not in the strong sense. A set S is *disconnected in the weak sense* by one of its subsets T, in case $S-T$ is not connected in the strong sense; it is *disconnected in the strong sense* in case $S-T$ is not connected even in the weak sense. Thus every set T which disconnects S in the strong sense also disconnects it in the weak sense but the converse is not true. Thus the set K', composed of the above set K together with the y-axis between $(0, +1)$ and $(0, -1)$ is disconnected in the weak sense by the removal of the origin but is not so disconnected in the strong sense. It is interesting to note that according to a theorem of R. L. Wilder, (1) if one limits himself to separating sets T which are closed, then if S is a continuous curve, T disconnects S in the weak sense only if it disconnects it in

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